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OPTIMIZATION OF RESERVOIR WATERFLOODING WITH UNSTABLE DISPLACEMENT FRONT

Shakhverdiev A.Kh.[1], Shestopalov Y.V.[2], Mandrik I.E.[3], Arefyev S.V.[4]

[1]Russian State Geological Prospecting University, Russia
23, Miklukho-Maklay str., Moscow, 117997
[2]Academy of Technology and Environment, University of Gävle,
Gävle, SE-801 76 Sweden
[3]PJSC LUKOIL Russia
11, Sretensky Boulevard, Moscow, 101000
[4] LUKOIL-Western Siberia LLC, Russia

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Summary. Non-stationary flooding of oil-saturated reservoirs has a long-standing and durable place as the main secondary method of oil production and maintenance of reservoir pressure in the development of most oil reservoirs. The water injection into the reservoir creates a delayed problem – the inevitable, often catastrophic flooding of oil production wells, provoked by a sudden and irreversible change in water saturation. The theory of two-phase flow filtration created by Buckley and Leverett does not take into account the loss of stability of the displacement front, which provokes an abrupt change and a triplicity of the water saturation value. Therefore, a mathematically simplified approach was proposed at one time, a repeatedly differentiable approximation to exclude a "jump" in water saturation. Such a simplified solution led to well-known negative consequences of the waterflooding practice, which experts call the "viscous instability of the displacement front", the "fingering displacement front". This work has presented a novel approach to formulation decisive rules for the first time allowing timely detection and prevention of the consequences of loss of stability of the displacement front and targeted control of the flooding system by stopping, forcing, limiting operating modes, assigning workover solutions of producing and injection wells. It is possible to quickly solve important short-term practical tasks passing traditional labor-intensive incorrect deterministic tasks and complex methods of solution mobilizing the injected water and controlling the fluid production rate, more precisely water and oil on the basis of the discriminant criterion.

1. Introduction

Conceptual foundations of the new paradigm. Several theoretical and applied problems remain outstanding in the field of scientific research on reservoir management with the dominance of a deterministic approach to research in recent years. Water flooding improvement under instability of the displacement front is most complex and important problems, and therefore, it has won the recognition of specialists as the main secondary method of oil recovery and enhanced oil recovery technology. Water flooding has become widespread primarily because of the manufacturability, availability, and inexpensiveness of water for injection into formations. However, this method does not provide the high-design planned indicators of the oil field development process.

The quality and quantity of studies related to the reservoir water flooding problems has increased significantly after the widespread use of geological and hydrodynamic mathematical modeling and its truncated modification including proxy models, sector and other simplified versions of model solutions but the shortcomings and imperfections inherent in the traditional approach remain.

The multidimensional geological and hydrodynamic models of multiphase flow solve the main long- and medium-term problems of oil and gas-saturated reservoir management including water flooding as a secondary method of increasing oil recovery. At the same time, important short-term operational flooding tasks cannot be solved using models, the main purpose of which is to calculate options for rational reservoir development for the purposes of drafting project documentation.

There is also an unsolved scientific and methodological problem of the Buckley-Leverett theory, which is used to mathematical formulation the flooding process in all existing hydrodynamic simulation software, the presence of a "jump" in water saturation and the triplicity of its value when oil is displaced by water are not taken into account. Ignoring this fact significantly reduces the efficiency of the water flooding, results in disruption of the optimal reservoir pressure maintenance, premature flooding of production wells is provoked, stagnant and poorly drained oil saturated zones are formed (Brouwer, Jansen, 2002; Sharma et al., 2020).

There are more than enough reasons for concern about the negative consequences due to the jump and triplicity of the water saturation value obtained in the Buckley-Leverett theory, some authors in their critical assessments regard the results as "absurd", and the lack of an exhaustive solution to the problem is an "intellectual dead end" (Buckley, Leverett, 1942; Rodygin, 2012). Anyway, at this stage, it is not possible to achieve a full-fledged unification of the waterflooding technology, since the two-phase flow theory based on the Buckley-Leverett theory does not enable the influence of the displacement front instability and, most importantly, timely preventing its negative consequences, which provoke an abrupt change in water saturation, which is fraught with catastrophic water breakthroughs to production wells (Duan et al., 2015).

The further fate of this «jump» in water saturation caused serious collisions among specialists. As a means of overcoming problems, it was proposed to calculate phase permeability based on water saturation with mathematically smooth approximations. However, it turned out that the simplified mathematical approximation does not correspond to an adequate physical mechanism of stability loss by the displacement front formed under rough conditions of interaction of hydrodynamic, capillary, molecular, inertial and gravitational forces.

Recently, there have been numerous laboratory and field studies, numerical experiments aimed at solving theoretical and practical problems of optimizing the process of non-stationary waterflooding, in which the above problem is confirmed (Baryshnikov et al., 2017; Brouwer, Jansen, 2002; Dake, 2001; Klebanov et al., 1982; Nigmatulin et al., 2014; Шахвердиев, Шестопалов, 2019; Shakhverdiev et al., 2019, 2021; Шахвердиев, Арефьев, 2021).

In an experimental study (Rodygin, 2012) of the dynamics of water content in bulk oil-saturated samples using the method of filtration waves of pressure (FWP) in relation to the basic provisions of the Buckley-Leverett theory, the author concludes that "an increase in the amplitude of pressure waves leads to an increase in the water content rate of the sample", that is, an increase in water saturation.

A number of authors try to solve the problem of the "jump" and the triplicity of water saturation using the fractal geometry of the structure of the oil displacement front (Baryshnikov et al., 2017; Ding et al., 2020; Duan et al., 2015). An attempt is made to explain the mechanism of oil displacement according to the principle of harmony of the flow structure (Baryshnikov et al., 2017). Based on the Hele-Shaw model in the experimental work (Baryshnikov et al., 2017), the development of instability of the displacement front of a viscous fluid from a porous medium in the form of jets when displaced by a fluid having a lower viscosity was studied. A change in the proportion of the volume of the pore space involved in jets of the displacing fluid was shown under oil displacement. The phenomenon of an abrupt change in the proportion of the total volume of the displacing jet in the volume of the pore space of the array at the leading edge of the displacement front was discovered and described at the viscosity ratios of liquids much smaller than one (Baryshnikov et al., 2017).

The authors of the study (Udy et al., 2017) note under implementation of the reservoir waterflooding that the gained experience shows that traditional and generally accepted approaches to waterflooding do not work here.

Nevertheless, the experience of applying new modifications of waterflooding technologies, which are usually used as the main method of reservoir pressure maintenance, as well as a method of enhanced and improved oil recovery in the development of oil and gas fields with scientifically based application, contains hidden potential.

Craig F.F. states according to the Buckley-Leverett equations, the conclusion follows that the velocity should have been the same for two different saturations, i.e. such saturation should be at a given point in the reservoir at the same time. To show even greater absurdity of this assumption, it should be noted that if the initial saturation gradient arose ahead of the displacement front prior to water injection, then we will obtain three different saturation values in the reservoir area according to the calculated data. Buckley and Leverett realized the physical impossibility of such a situation. They noted that the real saturation curve undergoes a discontinuity. Due to the three-valued saturation distribution, some researchers hesitated to apply the frontal displacement equation (Craig, 1971).

Dake L.P. expressed the problem as follows: "According to literary sources, such a triple value of saturation has been a certain intellectual problem for reservoir engineers for many years. Buckley and Leverett themselves claim that the triple value could have been avoided if they had been able to take capillary effects into account in their theory. Others suggested that this could be avoided by using a special method for solving differential equations known as the method of characteristic functions" (Dake, 2001).

Isaak Charnyi believed that "…starting from a certain point in time, the saturation distribution may turn out to be multi-valued, for example, similar to Riemann waves of finite amplitude, which are studied in the theory of shock waves. Obviously, the ambiguity of S_B (water saturation) is physically impossible" (Чарный, 1963).

Robert Nigmatulin actually confirms the same thesis and recommends looking for a solution in the field of the Riemannian compression wave in gas: "The function F_w' is nonmonotonic. During the evolution of the initial distribution of water saturation, when $\frac{d^2F}{dx^2} < 0$ occurs everywhere, the solution is obtained transferring along the characteristics (in a kinematic wave) of the initial distribution of water saturation. If there are zones where $\frac{d^2F}{dx^2} > 0$, then there is a "tipping" of the wave, similar to the tipping of the Riemann compression wave in gas, when large disturbances propagate at a higher velocity compared to weaker ones, which leads to the ambiguity of water saturation. The absurdity of such solutions led to the need to introduce surface or discontinuity (jump) of wave parameters (Klebanov et al., 1982; Nigmatulin et al., 2014).

Finally, there were attempts to find a numerical solution to the problem in hydrodynamic simulation of two-phase flow filtration in a porous medium. Aziz H., Settari E. came to the conclusion that: "Although both types of approximation (conductivity), as will be shown below, are second-order approximations, they lead to erroneous results. This is illustrated for the numerical solution of the Buckley-Leverett problem. With insignificant P_c (capillary pressure) in the *SS*-method equation, we obtain an almost hyperbolic problem with the correct solution, very close to the Buckley-Leverett solution. However, when grinding the mesh, the numerical results obtained by the "weighing" scheme differ from the real ones" (Aziz, Settari, 1986).

Similar opinions were expressed by many well-known scientists and specialists, and recommended looking for a solution to this complex problem in another field of mathematics (Шахвердиев, Шестопалов, 2019; Шахвердиев, 2004, 2017; Shakhverdiev et al., 2019, 2021; Шахвердиев, Арефьев, 2021; Wang et al., 2017).

It should be borne in mind that the engineering concepts that simplify and distort both the physical mechanism and the mathematical approximation of the water flooding parameters under conditions of instability of the displacement front lead to well-known theoretical solutions that differ greatly from the applied results. The well-known negative consequences of the waterflooding practice referred to experts as "viscous instability of the displacement front", "finger-shaped displacement front", "premature water breakthrough", "fractal geometry of the displacement front" indicate a low quality of flooding due to misunderstanding of the consequences of a simplified representation of the mechanism of the displacement process and ignoring adequate mathematical apparatus. Predicting and preventing the consequences of these catastrophic phenomena – surges in water saturation has become a very difficult task (Arnold et al., 2005; Skauge et al., 2009; Thompson, 1982). Therefore, it is extremely important to substantiate the physical mechanism of the formation and advancement of the displacement front and the early forecast of the growth or advancing rate of the water phase in the stream at certain sites and stages of the waterflooding (Мандрик и др., 2010; Мирзаджанзаде, Шахвердиев, 1997; Шахвердиев, Арефьев, 2021).

2. Problem statement

Let's consider the traditional joint isothermal flow of a two-phase liquid in a homogeneous porous medium as an argument in the absence of phase transitions by a system of differential continuity equations for each of the phases:

$$\left.\begin{aligned} \frac{\partial Q_W(x,t)}{\partial x} &= -m\frac{\partial \sigma_W}{\partial t} \\ \frac{\partial Q_O(x,t)}{\partial x} &= -m\frac{\partial \sigma_O}{\partial t} \end{aligned}\right\} \quad (1)$$

where Q_o, Q_W – are respectively accumulated oil and water extraction, σ_O, σ_W – are saturation of the medium with oil and water, *m* – is porosity.

In this case, from the ratio for the saturation $\sigma_O = 1 - \sigma_W$ we get

$$\left.\begin{aligned} \frac{\partial Q_W(x,t)}{\partial x} &= -m\frac{\partial \sigma_W}{\partial t} \\ \frac{\partial Q_O(x,t)}{\partial x} &= m\frac{\partial (1-\sigma_W)}{\partial t} \end{aligned}\right\} \quad (2)$$

Adding up the equations of system (2), we get:

$$\frac{\partial [Q_W(x,t)+Q_O(x,t)]}{\partial x} = 0$$

or

$$Q_W(x,t)+Q_O(x,t) \to Q_f(t). \quad (3)$$

The obtained result (3) shows that the volumetric flow rate $Q_f(t)$ does not depend on the spatial coordinate x, but depends only on time t. This does not mean that each individual phase also does not depend on the spatial coordinate. As a dynamic system, the dependence on time and on the spatial coordinate of the volume flow rate of each of the phases – oil and water – is of interest. Thus, the solution of system (2) does not allow estimating the dynamic behavior of each phase in the structure of a two-phase flow depending on the spatial coordinate x and the time t.

Having performed the known transformations of system (1) and solving the corresponding characteristic equation, we obtain (Abbasi et al., 2018; Чарный, 2006; Craig, 1971; Dake, 2001; Duan et al., 2015; Rose, Rose, 2004):

$$X_w = X_w(\sigma_w, 0) + \frac{V_w t}{m} f'(\sigma_w), \qquad (4)$$

where $X_w(\sigma_w, 0)$ is the initial distribution of water saturation at $t = 0$, m is porosity, V_w is the filtration rate of the aqueous phase, $f'(\sigma_w)$ is the derivative of the Buckley-Leverett function.

Fig. 1 shows the distribution of water saturation along the direction of the displacement front advance for different time points, obtained using the dependence (4). As can be seen from Fig. 1, there is a polysemy (triplicity) of the water saturation values at a certain point in time. Thus, attempts to obtain an acceptable result of solving the classical system of continuity equations and the filtration equation fail.

Fig. 1 shows the initial saturation field $s_0(x, 0)$ at $t = 0$ and the resulting $t > 0$ subsequent saturation field s_1; s_2; s_3. This is the simplest transformation of energy, consisting of merging and disappearance of the minimum and maximum under the action of a saturation jump forming one of the elementary catastrophes presented in Fig. 1, where the area is indicated in yellow and represents the zone of instability of the displacement front. According to the catastrophe theory, the resulting surface of the "fold" should be projected onto the plane of the control parameter, which divides this plane into an unstable one highlighted in yellow and the rest of the stable component as shown in Fig. 1. From a technological point of view, the instability predicted with a discriminant in water below zero combined with a discriminant in oil above zero is of interest, since this condition is a negative harbinger of a catastrophic breakthrough of water to producing wells.

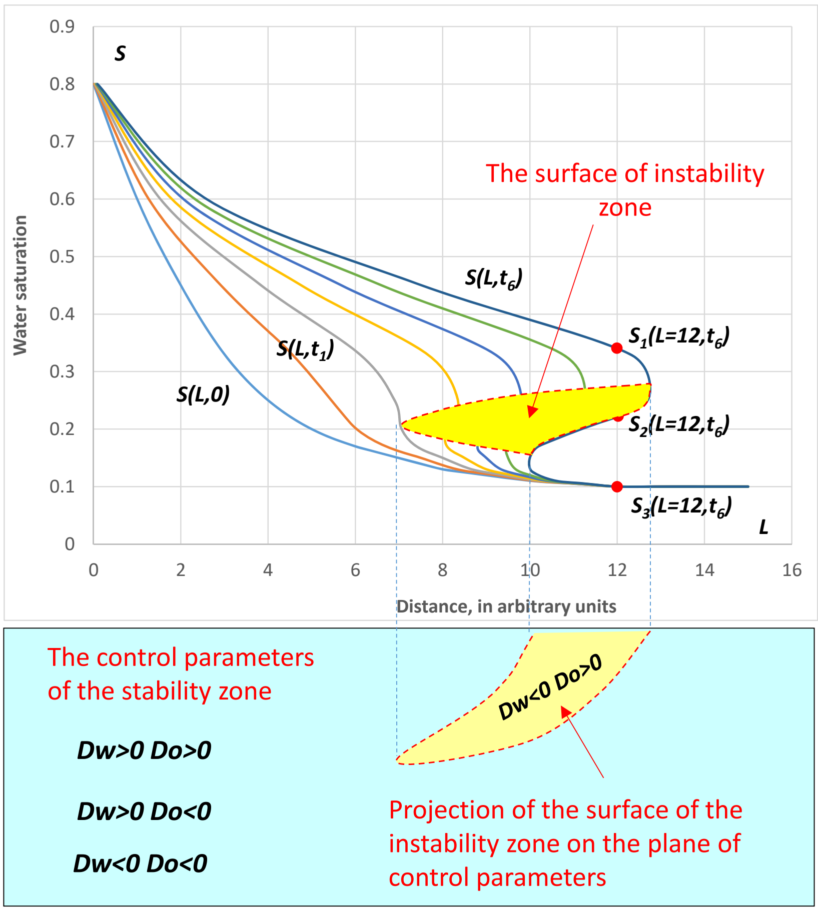

Fig. 1. The instability zone formed by the "jump" and the triplicity of water saturation

The above mentioned argument emphasizes the need to create a new scientific paradigm for the study of a dynamic multiphase filtration system designed to solve such operational tasks as an addition to the well-known geological and hydrodynamic models for the development of deposits of liquid and gaseous hydrocarbons.

It is fundamentally important to formulate and solve the problem of predicting the rate of change of the oil and water phases in the structure of the fluid flow leading to instability of the displacement front established on the basis of a control parameter developed in the theory of catastrophes calculated on the basis of the actual field data of wells operation obtained promptly. In this case, it is necessary to transform the actual data into certain new knowledge – unified control parameters that allow analyzing, monitoring and optimizing non-stationary flooding systems as provided for in the theory of catastrophes for discontinuous functions that change by leaps (Ermoliev et al., 2019).

It should be noted for the sake of objectivity that the mathematical apparatus known today as the "catastrophe theory" had not yet been developed at the time of the creation of the Buckley-Leverett theory, although the theory of Whitney singularities, the theory of Poincare bifurcations, and the basics of topology were well known to mathematicians (Gaiko, 2000). But the new mathematics was not popular in the engineering environment and this mathematical apparatus was not widely used in applied problems of technology.

The above mentioned theoretical prerequisites and argumentation indicate the need for an independent study of the dynamic behavior of the main indicators of the time development process.

3. Mathematical formulations

The qualitative theory of polynomial quadratic dynamical systems is the key to the formation of a discriminant criterion as a control parameter. The theory of catastrophes explores dynamical systems that make up a wide class of nonlinear systems including those described by quadratic polynomials.

The time series of the dynamics of current and accumulated oil and water withdrawals from the wells during the studied time period can also be described by a similar system of differential equations of growth models (Шахвердиев, Шестопалов, 2004, 2017; Shakhverdiev et al., 2019, 2021; Шахвердиев, Арефьев, 2021).

$$\frac{dQ_0}{dt} = a_0 Q_0^2 + b_0 Q_0 + c_0$$
$$\frac{dQ_w}{dt} = a_w Q_w^2 + b_w Q_w + c_w \qquad (5)$$

where $a_o, b_o, c_o, a_w, b_w, c_w$ are constant coefficients characterizing the behavior of the phases, Q_o and Q_w is the accumulated selection of oil and water, respectively.

It is assumed that system (5) describes the evolution in time at a given interval $t \in (t_0, t_1)$ and at given constant values of the parameters of the problem $a_o, b_o, c_o, a_w, b_w, c_w$, the accumulated production of oil $Q_o(t)$ and water $Q_w(t)$. In practice, the accumulated and current well selections are known, which makes it possible to determine the coefficients of the polynomials of the system (5) in the first approximation as constant values for a given time interval. For this purposes, it is necessary to qualitatively investigate the behavior of the solutions of system (5) using the mathematical apparatus of polynomial quadratic dynamical systems (Шахвердиев, Шестопалов, 2019; Shakhverdiev, Mandrik, 2007). To this end, it is necessary to consider the properties of the general solution of each of the equations of the system on the phase plane Q_o of t, depending on the parameters of the problem. It is also necessary to consider the behavior and properties of solutions of the system on the phase plane Q_o, Q_w based on all parameters of the problem.

The creation of a qualitative theory that includes a description of all special and critical points of an autonomous symmetric polynomial dynamical system and the construction of the entire variety of its phase portraits is quite a difficult task. To do this, it is necessary to obtain general solutions and solutions to Cauchy problems in all possible classes defined by various types of factorizations, which as will be shown are determined by a combination of signs and absolute values of discriminants of polynomials, and to identify their most important properties.

It should be noted that, in general, polynomial dynamical systems are not integrable and the description of each of their new integrable families is an important independent contribution to the theory of dynamical systems. In particular, even two-dimensional systems which right-hand sides are polynomials of the second degree of two variables are not integrable, and the theory of such systems is far from complete (Barenblatt et al., 2003; Gaiko, 2000). Using the results and the integration technique (Шахвердиев, Шестопалов, 2019; Shakhverdiev et al., 2014; Shakhverdiev et al., 2022), we show that system (5) is integrable into elementary functions. This fact is important (a) both from a theoretical point of view, since a new class of integrable dynamical systems is fully described, and (b) from a practical one, since it makes it possible to build a complete qualitative theory and obtain –

without the use of numerical integration – all general and particular solutions of the system and the corresponding problems with initial conditions (Cauchy problem).

Carrying out a qualitative analysis of two-dimensional APDS in this paper, we provide a description and visualization of various scenarios of models of specific processes using the proven integrability (5) in elementary functions. The obtained results are generalized for the first time in a compact matrix and criterion form, which allows determining various types of all rest points of the system (5), as well as its stable and unstable solutions and their singular points depending on the signs of the discriminants of the polynomials. The latter is of decisive importance for practical applications of the results of the qualitative theory constructed in the work.

In order to conduct a complete qualitative study of the solutions of an autonomous system including the analysis of all its possible critical and singular points and solutions, we consider the behavior of the solutions of the autonomous system (5) and the corresponding equation on the phase plane (Q_o, Q_w) depending on the parameters of the problem.

We assume that the system (5) and the equations derived from it

$$\frac{dQ_W}{dQ_O} = \Phi(Q_O, Q_W),\ \ \Phi(Q_O, Q_W) = \frac{f_W(Q_W)}{f_O(Q_O)}, \quad (6)$$

describe the dynamics of changes in Q_w depending on Q_0. This dynamics is obtained in the form of solutions of equation (6) under various "initial" conditions $Q_W(Q_O^0) = Q_W^0$ and corresponding curves $Q_W = Q_W(Q_O)$ on the phase plane (Q_O, Q_W).

In order to construct a qualitative theory of polynomial quadratic DS and introduce a discriminant criterion of qualitative theory, we concretize, following (Шахвердиев, Шестопалов, 2019), the definitions of classes D^{++}, D^{-+}, D^{+-} and D^{--} as sets $\{f_o, f_w\}$ of pairs of polynomials $f_o(Q_O) = a_o {Q_O}^2 + b_o Q_O + c_0$ and $f_w(Q_w) = a_w {Q_w}^2 + b_w Q_w + c_w$ such that

$$D^{++}: D_o = {b_o}^2 - 4a_o c_o > 0$$

and

$$D_w = {b_w}^2 - 4a_w c_w > 0;$$

$$D^{-+}: D_o < 0 \text{ и } D_w > 0;$$
$$D^{+-}: D_o > 0 \text{ и } D_w < 0;$$
$$D^{--}: D_o < 0 \text{ и } D_w < 0.$$

Using integration methods and formulas (Шахвердиев, Шестопалов, 2019; Shakhverdiev, 2019; Shakhverdiev, 2004; Шахвердиев, 2017), we will summarize in a table (Table) all explicit formulas for the general solution of equation (6) with respect to phase variables (Q_O, Q_W) for classes $D^{\pm}$ with nonzero discriminants, as well as for classes $D^{0\pm}, D^{\pm 0}$, when one of the discriminants turns to 0.

General solutions of the equation $\frac{dQ_W}{dQ_O} = \frac{a_W {Q_W}^2 + b_W Q_W + c_W}{a_O {Q_O}^2 + b_O Q_O + c_0}$ in classes $D^{\pm},\ D^{0\pm},\ D^{\pm 0}$

№№	Classes	Example of the equation	**General solutions** $\frac{dQ_W}{dQ_O} = \frac{a_W {Q_W}^2 + b_W Q_W + c_W}{a_O {Q_O}^2 + b_O Q_O + c_0},\ Q_O^0 = \frac{b_O}{2a_O},\ Q_W^0 = \frac{b_W}{2a_W}$
1	D^{++}	$\frac{dQ_w}{dQ_o} = \frac{Q_w(Q_w - 1)}{Q_O(Q_O - 1)}$	$Q_W(Q_O;C) == Q_W^1 - \frac{\sqrt{D_W}}{a_W\left(-1 + C\left\|\frac{Q_O - Q_O^2}{Q_O - Q_O^1}\right\|^{\rho}\right)}$, $\rho = \sqrt{\frac{D_W}{D_O}} > 0.$
2	D^{++}	$\frac{dQ_w}{dQ_o} = \frac{{Q_w}^2}{Q_O(Q_O - 1)}$	$Q_W(Q_O;C) = Q_W^0 + \frac{1}{\frac{a_w}{\sqrt{D_o}} \ln\left\|\frac{Q_O - Q_O^2}{Q_O - Q_O^1}\right\| + C}$
3	D^{++}	$\frac{dQ_w}{dQ_O} = \frac{Q_w(Q_w - 1)}{{Q_O}^2}$	$Q_W(Q_O;C) = Q_W^1 + \frac{\sqrt{D_w}}{a_w}\frac{1}{1 + Ce^{\frac{\sqrt{D_W}}{a_O(Q_O + Q_O^0)}}}$
4	D^{++}	$\frac{dQ_w}{dQ_o} = \frac{{Q_w}^2}{{Q_O}^2}$	$Q_W(Q_O;C) = Q_W^0 - \frac{1}{\frac{a_w}{a_o(Q_O^0 - Q_O)} + C}$

№№	Classes	Example of the equation	General solutions $\frac{dQ_W}{dQ_O}=\frac{a_W{Q_W}^2+b_WQ_W+c_W}{a_O{Q_O}^2+b_OQ_O+c_0},\ Q_O^0=\frac{b_O}{2a_O},\ Q_W^0=\frac{b_W}{2a_W}$
5	D^{++}	$\frac{dQ_w}{dQ_O}=\frac{{Q_w}^2+1}{Q_O(Q_O-1)}$	$Q_W(Q_O;C)=-Q_W^0+P\tan\left(\frac{\rho}{2}\ln\left\|\frac{Q_O-Q_O^1}{Q_O-Q_O^2}\right\|+C\right)$, $\rho=\sqrt{-\frac{D_w}{D_o}}>0, P=\frac{\sqrt{-D_w}}{2a_w}$
6	D^{++}	$\frac{dQ_w}{dQ_O}=\frac{Q_w(Q_w-1)}{{Q_O}^2+1}$	$Q_W(Q_O;C)=Q_W^1-\frac{\sqrt{D_w}}{P}\frac{1}{-1+Ce^{-2\rho\tan^{-1}\left(\frac{Q_O^0+Q_O}{P}\right)}}$, $\rho=\sqrt{-\frac{D_w}{D_o}}>0, P=\frac{\sqrt{-D_o}}{2a_o}$
7	D^{++}	$\frac{dQ_w}{dQ_O}=\frac{{Q_w}^2+1}{{Q_O}^2+1}$	$Q_W(Q_O;C)==-Q_W^0+P_2\tan\left(\rho\tan^{-1}\left(\frac{Q_O^0+Q_O}{P_1}\right)+C\right)$, $\rho=\sqrt{\frac{D_w}{D_o}}>0,\ P_1=\frac{\sqrt{-D_o}}{2a_o},\ P_2=\frac{\sqrt{-D_w}}{2a_w}$
8	D^{++}	$\frac{dQ_w}{dQ_O}=\frac{{Q_w}^2+1}{{Q_O}^2}$	$Q_W(Q_O;C)=Q_W^0+P\tan\left(\frac{\sqrt{-D_w}}{2}\left(\frac{1}{a_o(Q_O^0-Q_O)}+C\right)\right)\ P=\frac{\sqrt{-D_w}}{2a_w}$
9	D^{++}	$\frac{dQ_w}{dQ_O}=\frac{{Q_w}^2}{{Q_O}^2+1}$	$Q_W(Q_O;C)=Q_W^0-\frac{1}{\frac{2a_w}{\sqrt{-D_o}}\tan^{-1}\left(\frac{Q_O^0+Q_O}{P}\right)+C}$, $P=\frac{\sqrt{-D_o}}{2a_o}$

4. Solutions analysis and visualization of phase portraits

The main property of a Dynamic System (DS) is its stability, which is used to assess how the system reacts to external influences when it is assumed that its current state remains unchanged. If disturbances increase over time, this leads to a loss of stability accompanied by a sharp (catastrophic) transition to a qualitatively new state.

A systematic study of the properties of solutions of DS (5) or Equation (6) allows drawing the following conclusions: the formation of catastrophic changes in solutions within various scenarios of the development of disasters occurs when the sign of one of the discriminants of square trinomials changes, that is, while moving from one class $D^{\pm}$ to another. The latter corresponds to the transition of the system described by DS from one state to a qualitatively different one. At the same time, each class corresponds to a special structure of decision curves. A change in this structure including a catastrophic one accompanied by the appearance of new types of partial solutions that are absent in the "previous" class occurs when one parameter is varied – the discriminant D_O or D_W of one of the three terms. The D^{++} class plays a special role as the 'source' of such changes, since, as it was shown, solutions have the largest number of critical points (four).

Transitions $D^{++}\to D^{0+}$ or $D^{++}\to D^{+0}$ from class D^{++} to class D^{0+} or D^{+0} take place when the discriminants of the square trinomials of equation (5) $D_o={b_o}^2-4a_oc_o={a_o}^2(Q_O^1-Q_O^2)^2>0$ or $D_w={b_w}^2-4a_wc_w={a_w}^2(Q_W^1-Q_W^2)^2>0$ (for $a_o, a_w\neq 0$) in class D^{++} tends to 0, which is equivalent to the limit transition $Q_O^2-Q_O^1\to 0+(Q_O^2>Q_O^1)$ or $Q_W^2-Q_W^1\to 0+(Q_W^2>Q_W^1)$. The latter means that the distance between the Q_o or Q_w – coordinates of the critical points tends to 0, that is, the critical points converge and merge along one of the sides of the rectangle at the vertices of which they are located (in the D^{++} class).

Below are fragments of a series of phase portraits that characterize all the most important transitions between classes and scenarios for the development of disasters as a result of variation of one of the discriminants.

Transition from class D^{++} to class D^{+-} through classes D^{+0}, D^{00}, D^{0-}

A. Transition from class D^{++} to class D^{+0}

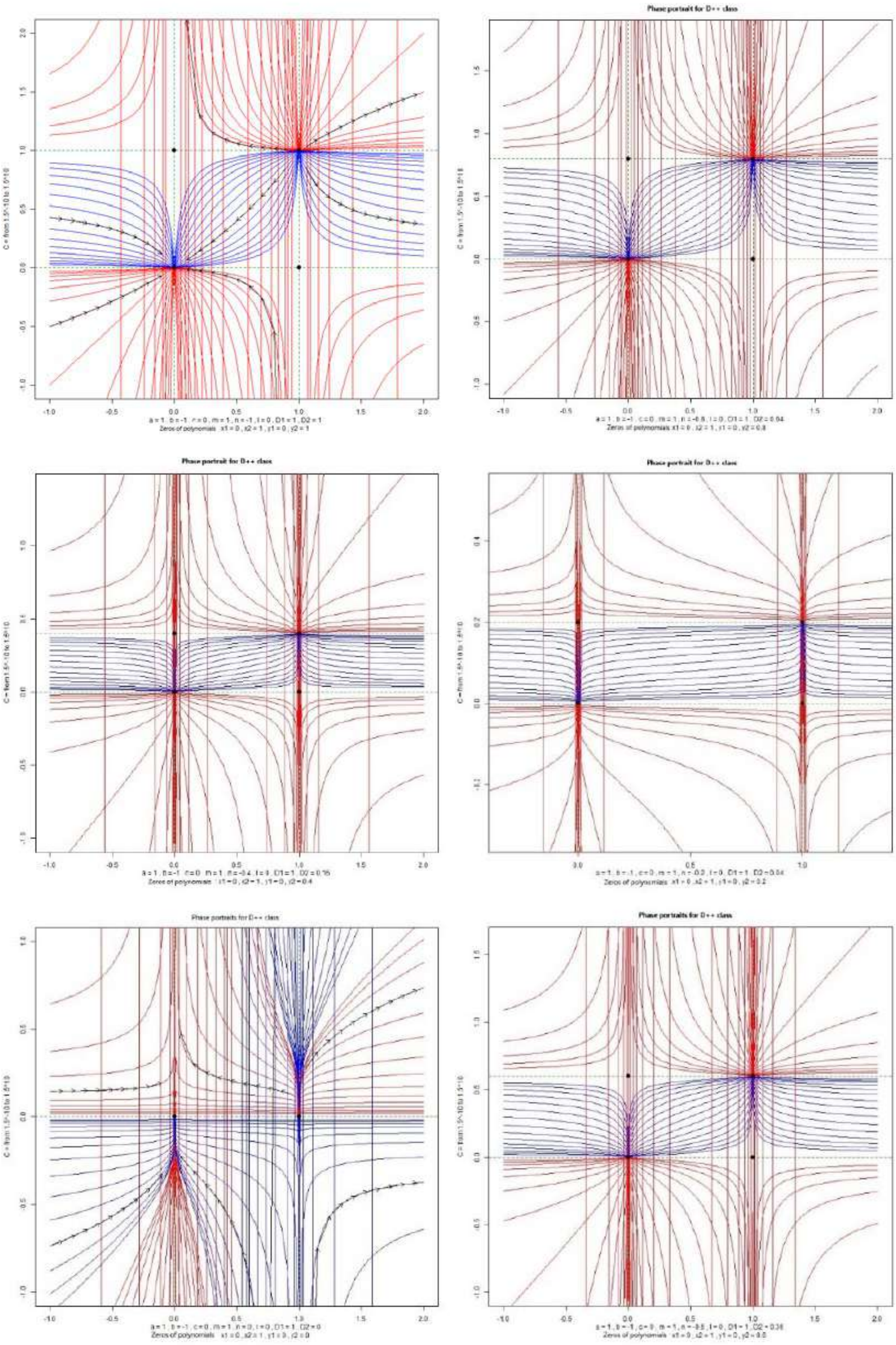

Fig. 2. Phase portraits during the transition from class D^{++} to class D^{+0}

The approach to the transition from class D^{++} to class D^{00} through D^{+0} class, that is, the convergence and merging of nodal and saddle singular points, which are located on the left and right sides of the rectangle in Fig. 2a-2d occurs with a decrease and a tendency to 0 of the discriminant D_w (D_w = 1, 0.64, 0.36, 0.16, 0.04 in Fig. 2a-2d, respectively), so, that the distance between the Q_w coordinates of the singular points tends to 0. When the = 0, in D^{+0} class there is a merger (a) of two singular nodal and saddle points 1 and 3 (the left two in Fig. 2a-2d) as a result of which one point 1 is formed (in Fig. 2e) is a stable "one-sided" node, and (b) two nodal and saddle singular points 2 and 4 (the right two in Fig. 2a-2d), resulting in one point 2 – an unstable "one-sided" node (Fig. 2e). A gap is also formed at D_w =0 and a discontinuous "fold" characteristic of class D^{+0}.

5. Visualization and analysis of solutions curves to Cauchy problems

Solutions of the Cauchy problems are particular solutions, that is, solutions obtained from the general solution formula for a specific value of an arbitrary constant that corresponds to the initial conditions set. Such solutions pass through a given point (Q_O^0, Q_W^0) of the phase plane (Q_O, Q_W) and the local uniqueness theorem of the solution is fulfilled.

Class D^{+-} if $a_O = 1, b_O = 0, c_O = -0.25$, $D_\text{o} = 1$; $a_W = 1$, $b_W = 2$, $c_W = 1.25$, $D_\text{W} = -1$ for each point of the plane (Q_O^0, Q_W^0) with $Q_O^0 \neq \pm 0.5$, $Q_O \neq \pm 0.5$ and

$$C = C_0 = C(Q_O^0, Q_W^0) = \tan^{-1}(2Q_W^0 + 2) - \\ -\frac{1}{2}\ln\left|\frac{Q_O^0+0.5}{Q_O^0-0.5}\right|, \quad Q_O^0 \neq \pm 0.5 \tag{8}$$

Function

$$Q_W(Q_O, C_0) = -1 + \frac{1}{2}\tan\left(\frac{1}{2}\ln\left|\frac{Q_O+0.5}{Q_O-0.5}\right| + \\ +\tan^{-1}(2Q_W^0 + 2) - \frac{1}{2}\ln\left|\frac{Q_O^0+0.5}{Q_O^0-0.5}\right|\right) \tag{9}$$

is a solution to the Cauchy problem

$$\frac{dQ_W}{dQ_O} = \frac{{Q_W}^2 + 2Q_W + 1.25}{{Q_O}^2 - 0.25}, \qquad Q_W(Q_O^0) = Q_W^0.$$

At least two scenarios of the behavior of solutions to the Cauchy problem are possible:

(a) at $-0.5 < Q_O^0 < 0.5$ and any Q_W^0 are monotonically increasing functions that lie in the band between two special stationary solutions

$$Q_O = -0.5, Q_O = 0.5;$$

(b) at $Q_O^0 > 0.5$ or $Q_O^0 < -0.5$ and any Q_W^0 are monotonically decreasing unbounded functions that lie outside the band between two stationary solutions. In this case, the entire phase plane is filled with solution curves (Fig. 3).

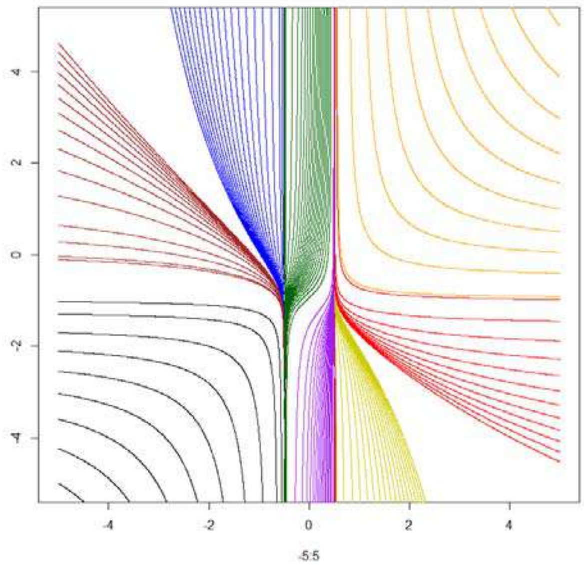

Fig. 3. Graph of curves of solutions of the Cauchy-function problem (9) for D+-. Black and blue lines at *C*= from -1.4652 to -0.4388, brown lines at *C*=from 1.1611 to 1.5823, dark yellow and orange lines at *C*=from 0.0689 to 1.4652, red lines at *C*=from-1.43 to 0.05, purple lines at *C*= from 1.1866 to 1.97, green lines at *C*= from 0.129 to 1.026

For D^{-+} class function

$$Q_W(Q_O) = \frac{1}{1 + C_0 e^{\tan^{-1} Q_O}} = \\ = \frac{1}{1 + \left(1 - \frac{1}{Q_W^0}\right) e^{\tan^{-1} Q_O - \tan^{-1} Q_O^0}}$$

is for $Q_W^0 \neq 0$ the solution of the Cauchy problem

$$\frac{dQ_W}{dQ_O} = \frac{Q_W(Q_W-1)}{{Q_O}^2+1}, \; Q_W(Q_O^0) = Q_W^0.$$

Thus, $C_0 < 0$ is performed for $0 < Q_W^0 < 1$, and therefore, the curves of solutions to the Cauchy problems are, for any values of Q_O^0, graphs of bounded monotone functions that all lie in the band between two stationary solutions $Q_W = 0, Q_W = 1$. If $Q_W^0 > 1$, then the curves of solutions to the Cauchy problems are, for any values of Q_O^0, graphs of the unlimited monotone functions that lie outside the band between two stationary solutions as it is seen in Fig. 4 and 5. We can conclude about at least two possible scenarios of the behavior of the curves of solutions to the Cauchy problems:

(a) for $0 < Q_W^0 < 1$ and any Q_O^0: monotonically decreasing functions that lie in the band between two stationary solutions $Q_W = 0, Q_W = 1$;

(b) for $Q_W^0 > 1$ or $Q_W^0 < 0$ and any x_0: monotonically increasing unbounded functions that lie outside the band between the lines $Q_W = 0, Q_W = 1$.

In this case, the entire phase plane is filled with solution curves.

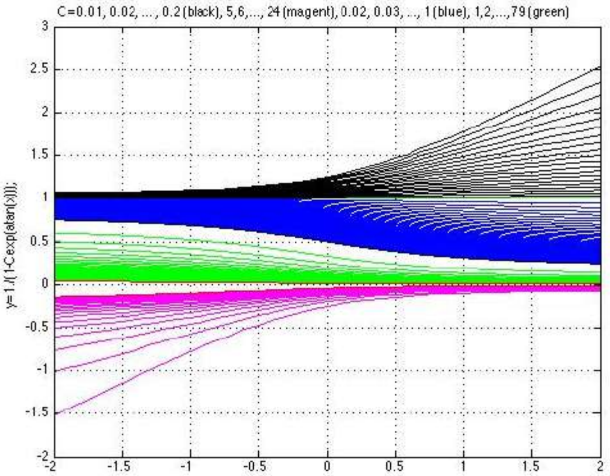

Fig. 4. General solution of D^{-+} class $Q_w(Q_O; C) = \frac{1}{1-Ce^{\tan^{-1}Q_O}}$, $C = -0.02, -0.04, \ldots, -1$ (blue curves), $-2, -3, \ldots, -79$ (green curves), $0.02, 0.04, \ldots, 0.2$ (black curves), $5, 6, \ldots, 24$ (purple curves)

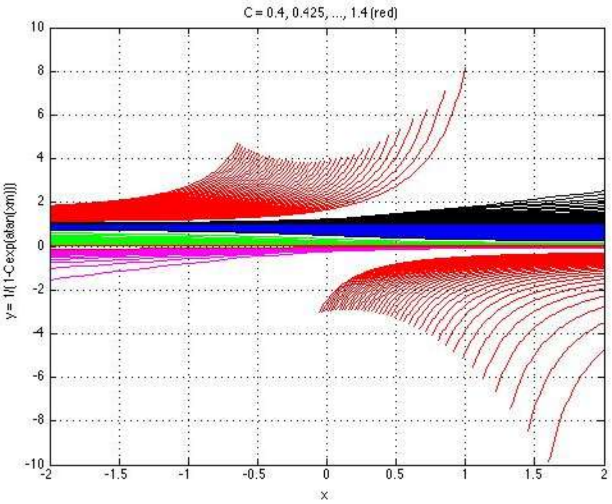

Fig. 5. General solution of D^{-+} class $Q_w(Q_O; C) = \frac{1}{1-Ce^{\tan^{-1}Q_O}}$, $C = -0.02, -0.04, \ldots, -1$ (blue curves), $-2, -3, \ldots, -79$ (green curves), $0.02, 0.04, \ldots, 0.2$ (black curves), $5, 6, \ldots, 24$ (purple curves), $0.4, 0.425, \ldots, 1.4$ (red curves of solutions with movable singular points)

The study of the properties of the general solutions of system (5) or equation (6) in all discriminant classes allows concluding about the exceptional richness of the qualitative diversity of these solutions. They include, in particular, all logistics-type scenarios (as a subset of D^{++} class solutions), as well as many other previously undescribed scenarios and types of transient disasters. This work is the first introductory study that provides the scientific community with a comprehensive study and application of this new class of integrable DS.

6. Application of results of qualitative analysis to solve the inverse problem

The practice of using a discriminant criterion to predict a water breakthrough is the following. It is necessary to determine the relationship between the rate of change of the measured values and these values themselves. The possibility of the above mentioned classification and qualitative analysis of solutions of equations in the time domain and autonomous equations is based on the known relationships between the rate of change of the measured quantities and these quantities themselves. These relations have the form of ordinary differential equations (6) and (7) resolved with respect to the derivative.

Using the Least Squares Method (LSM), it is possible to approximate by a quadratic polynomial the right part of the system (6) – the magnitude of the change in the measured value per unit of time (velocity) $v_{oi} = \frac{Q_{oi+1} - Q_{oi}}{t_{oi+1} - t_{oi}}$, $v_{wi} = \frac{Q_{wi+1} - Q_{wi}}{t_{wi+1} - t_{wi}}$, $v_{oi} \approx \frac{dQ_o}{dt}$, $v_{wi} \approx \frac{dQ_w}{dt}$, to get the ratio (6), more precisely, to solve the inverse problem – to determine the coefficients a_O, b_O, c_O, a_W, b_W, c_W using LSM, provided that the values of Q_{oi} and Q_{wi} i=1, 2 $i = 1, 2, \ldots n$ are known.

The above mentioned leads to the conclusion that the set of catastrophes of the growth equation is determined by the set of solutions obtained when the discriminants for oil and water are equal to zero. The resulting conclusion for the problem of hydrodynamic effect will take the following form:

$$D_O = {b_O}^2 - 4a_O c_O = 0$$
$$D_W = {b_W}^2 - 4a_W c_W = 0,$$

where the coefficients a_O, b_O, c_O, a_W, b_W, c_W for each oil or water phase are determined from equation (6) by the well-known least squares method (LSM), and the resulting system of algebraic equations is solved by the Gauss method.

The discriminant criterion and the corresponding strategies for choosing the operating mode of the well can be formulated as follows:

- when $D_O < 0$ and $D_W > 0$ (the family of solutions D^{-+} from Table), the selection of oil has a trend to increase, and water – to decrease, it is recommended to increase the selection of liquid from the well while studying the potential of pumping equipment;
- when $D_O > 0$ and $D_W < 0$ (the family of solutions D^{+-} from Table), the extraction of oil has a downward trend, and water – to increase, while a water breakthrough is possible, it is recommended to limit the extraction of liquid from the well or taking geological and technical operations on water shut-off;

• when $D_O < 0$ and $D_W < 0$ (the family of solutions D^{--} from Table), the oil and water withdrawals have an upward trend, it is recommended not to change the operating mode of the well and to perform geological and technical operations on water shut-off;

• when $D_O > 0$ and $D_W > 0$ (the D^{++} family of solutions in Table), oil and water withdrawals have a downward trend, it is recommended to carry out geological and technical operations on well stimulation.

An example of using a discriminant criterion for the development data of a specific object with the possibility of predicting the instability of the displacement front at a specific well is given below. According to the proposed methodology, the red line in Fig. 6(b) corresponds to the zero value of the discriminant, and this line determines the trend on the positive or negative side of the discriminant for water and oil, which occurs while the oil displacement front is unstable with water. Depending on the combination of the absolute value and the sign of the discriminant, the unstable state of the displacement front and the expected water breakthrough to the producing well are predicted in advance for three months, April 04.2019.

As it can be seen in Fig. 6, a water breakthrough has occurred since 04.2019 according to the discriminant criterion, this is confirmed by $D_o > 0$ $D_w < 0$ and is predicted three months before the water breakthrough while crossing the red line. At the same time, it is not possible to determine the instability of the displacement front and predict the water breakthrough in advance based on the dynamics of oil and water extraction in Fig. 6(a) on April 4, 2019. In this case, it is recommended to reduce the selection of fluid through the well, determine the cause of a possible water breakthrough, and take appropriate geological and technical measures, in particular, repair and insulation work to limit the growth of the water content of the well products. It is recommended to reduce the injection of water through the surrounding interacting injection wells for this period.

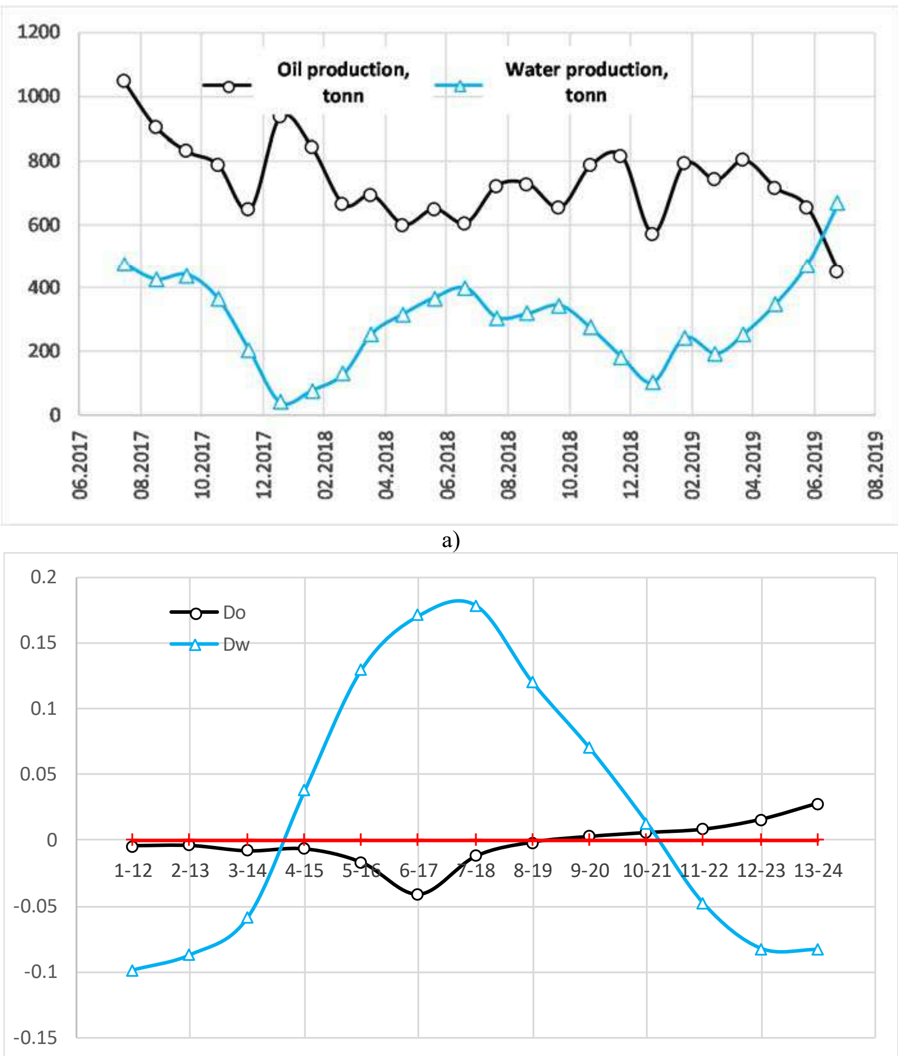

Fig. 6. Dynamics of oil and water production (a) and dynamics of discriminants (b) for well #3166G

It should be noted that the zone of instability of the displacement front on the discriminant plane corresponds to the state of instability in Fig. 6(b) starting from point 10-21, where D_w passes into the region of positive values (decrease), and D_w into the region of negative values (growth). As new data become available, further calculations are carried out in interactive mode according to the methodology for the subsequent specification of recommendations to optimize the operating modes of the wells.

7. Conclusions

The proposed new solutions to the direct and inverse problems of the catastrophe theory allow considering the consequences of instability of the displacement front in a timely manner and predicting the consequences of abrupt change and triplicity arising from the Buckley-Leverett theory.

A complete qualitative theory of such dynamical systems is constructed including an exhaustive analysis of all their singular points and features of solutions. The properties of solutions in the phase planes of parameters and in time are studied. It is shown that the discriminants of polynomials are the control parameters controlling the essential properties of solutions, a classification of solutions is proposed depending on the values and signs of the discriminants and belonging to a specific family D^{++}, D^{+-}, D^{-+} or D^{--}.

Decisive rules were formulated for the first time on the basis of the proposed criteria, allowing timely detection and prevention of the consequences of loss of stability of the displacement front and targeted regulation of the flooding system stopping, forcing, limiting operating modes, assigning well interventions to producing and injection wells. It is possible to quickly solve important short-term practical tasks passing traditional labor-intensive incorrect deterministic tasks and complex methods of solving them mobilizing the injected water and regulating the selection of liquid, more precisely water and oil, on the basis of the discriminant criterion.

It should be noted that the proposed methodology is sufficiently mobile and accurate for monitoring and controlling both the state of the flooding process and the well stock, and in general for the development of deposits in the "on-line" mode. There is a potential opportunity to replenish the database on a monthly basis based on the proposed criteria and decisive rules to make specific clarifications to the work program and geological and technical measures including while optimizing the reservoir pressure maintenance system by regulating the operating modes of producing and injection wells.

The proposed solutions are an integral part of the new concept of nonstationary flooding, which provides for an early forecast and taking measures in case of instability of the displacement front and, as a result, water breakthrough in producing wells.

The system optimization of flooding of oil fields is designed for a certain technological and economic effect of resource conservation and energy efficiency.

This opens up the possibility of system optimization of non-stationary flooding and the prospect of increased oil recovery and intensification of oil production, as well as effective mobilization of injected and extracted water and gas.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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ОПТИМИЗАЦИЯ ЗАВОДНЕНИЯ ПЛАСТОВ С НЕУСТОЙЧИВЫМ ФРОНТОМ ВЫТЕСНЕНИЯ

Шахвердиев А.Х.[1], Шестопалов Я.В.[2], Мандрик И.Е.[3], Арефьев С.В.[4]

[1]*Российский государственный геологоразведочный университет*
117997, Москва, ул. Миклухо-Маклая, д.23
[2]*Академия технологии и окружающей среды, Университет Эвле,*
SE-801 76 Швеция
[3]*ПАО ЛУКОИЛ, Россия*
101000, Москва, Сретенский бульвар,11
[4]*ООО «ЛУКОЙЛ-Западная Сибирь», Россия*

Резюме. Нестационарное заводнение нефтенасыщенных пластов давно и прочно занимает место основного вторичного метода добычи нефти и поддержания пластового давления при разработке большинства нефтяных залежей. Закачка воды в пласт создает отложенную проблему – неизбежное, часто катастрофическое обводнение нефтедобывающих скважин, спровоцированное резким и необратимым изменением водонасыщенности. Созданная Бакли и Левереттом теория фильтрации двухфазных потоков не учитывает потерю устойчивости фронта вытеснения, что провоцирует резкое изменение и утроенные значения водонасыщенности. Поэтому в свое время был предложен математически упрощенный подход – многократно дифференцируемая аппроксимация, исключающая "скачок" водонасыщенности. Такое упрощенное решение привело к хорошо известным негативным последствиям практики заводнения, которые специалисты называют "вязкостной неустойчивостью Фронта вытеснения", "пальцеобразным вытеснением". По результатам исследований предлагается полная качественная теория подобных квадратичных полиномиальных динамических систем, включающая исчерпывающий анализ всех их сингулярных точек и особенностей решения. Свойства решений изучались на фазовых плоскостях самих параметров и времени. Показано, что полиномиальные дискриминанты представляют собой управляющие параметры, определяющие существенные свойства решений. Предложена классификация решений в зависимости от знаков дискриминантов и от семейства, к которому принадлежит решение. В данной работе впервые представлен новый подход к формулированию решающих правил, позволяющих своевременно обнаружить и предотвратить последствия потери устойчивости фронта вытеснения и целенаправленно управлять системой заводнения путем остановки, форсирования, ограничения режимов работы, назначения ремонтных режимов добывающих и нагнетательных скважин. Появляется возможность оперативно решать важные краткосрочные практические задачи, минуя традиционные трудоемкие некорректные детерминированные задачи и сложные методы решения, мобилизуя нагнетаемую воду и управляя дебитом жидкости, точнее воды и нефти на основе дискриминантного критерия.

Ключевые слова: *заводнение, неустойчивость фронта вытеснения, оптимизация, «пальцеобразование», теория катастроф, фазовая плоскость*

QEYRİ-SABİT SIXIŞDIRMA CƏBHƏSİ İLƏ SU LAYLARIN SUVURMANIN OPTİMALLAŞDIRILMASI

Şahverdiyev A.X.[1], Şestopalov Y.V.[2], Mandrik I.E.[3], Arefev S.V.[4]

[1]*Rusiya Dövlət Geoloji Kəşfiyyat Universiteti, Rusiya*
117997, Moskva, Mikluxo-Maklay küç., 23
[2]*Texnologiya və Ətraf Mühit Akademiyası, Evle Universiteti,*
SE-801 76 İsveç
[3]*PJSC "Lukoil", Rusiya*
101000, Moskva, Sretensky Bulvarı, 11
[4]*«LUKOIL-Qərbi Sibir» MMC, Rusiya*

Xülasə. Neft yataqlarının işlənmə zamanı hasilatını və lay təzyiqini sabit saxlaması üçün neftlə doymuş laylara qeyri-stasionar suvurma texnologiyası çoxdan əsas ikinci üsul kimi qəbul olunur. Laya suyun vurulması su ilə doyma əmsalın kəskin və geri dönməz dəyişməsi səbəbindən quyuların qaçılmaz və katastrofik sulaşması təxirə salınmış bir problem yaradır. Buckley və Leverett tərəfindən yaradılan iki fazalı axınların filtrasiya nəzəriyyəsi, suyun doymasının kəskin dəyişməsinə və üçqat dəyərinə səbəb olan sıxışdırma cəbhəsinin sabitliyinin itirilməsini nəzərə almır. Buna görə, bir vaxtlar riyazi cəhətdən sadələşdirilmiş bir yanaşma təklif edildi – Su doymasının "sıçrayışını" istisna edən dəfələrlə fərqlənən yaxınlaşma. Belə bir sadələşdirilmiş həll, mütəxəssislərin "sıxışdırma cəbhəsinin viskoz qeyri-sabitliyi", "barmaq şəklində sıxışdırma" adlandırdığı suvurma təcrübəsinin mənfi nəticələrinə səbəb oldu. İşdə ilk dəfə sıxışdırma cəbhəsinin dayanıqlığının itirilməsinin nəticələrini vaxtında aşkar etməyə və qarşısını almağa və iş rejimlərini məhdudlaşdırmaqla, sürətləndirməklə, məhdudlaşdırmaqla, hasilat və vurucu quyularının təmir rejimlərini təyin edilməklə, suvurma sistemini məqsədyönlü şəkildə idarə etməyə imkan verən həlledici qaydaların formalaşdırılmasına yeni bir yanaşma təqdim edilir. Ənənəvi vaxt aparan korrekt olmayan deterministik məsələləri və mürəkkəb həll üsulları keçərək, vurulan suyu səfərbər edərək və diskriminant meyarı əsasında mayenin, daha doğrusu su və neftin axın sürətini idarə edərək vacib qısamüddətli praktik problemləri tez bir zamanda həll edilməsi mümkün olur.

Açar sözlər*: suvurma, sıxışdırma cəbhəsinin qeyri-sabitliyi, optimallaşdırma, "barmaq şəklində sıxışdırma", fəlakət nəzəriyyəsi, faza müstəvisi*